Divergence of the Floquet-Magnus expansion in a periodically driven energy bounded system

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Floquet theory

Periodically driven quantum systems:

$$H(t) = H_0 + \cos\omega t\, H_1, \qquad \omega = 2\pi/T$$

Time evolution operator:

$$i\partial_t U(t) = H(t)U(t), \qquad U(0) = \mathrm{I}$$

Floquet Hamiltonian H_F is defined by

$$e^{-iH_F T} = U(T)$$

Floquet-Magnus expansion:

$$H_F = \sum_{n=0}^{\infty} \Omega_n T^n$$

Conjecture

The energy of the system remains finite.
⇒ The FM expansion converges

The driven system indefinitely heats up.
⇒ The FM expansion diverges

Problem

Driven harmonic oscillator:

$$H(t) = \frac{1}{2}p^2 + \frac{1}{2}\omega_0^2 x^2 + gx\cos\omega t$$

The FM expansion converges for $T < \frac{2\pi}{\omega_0}$.

Driven anharmonic oscillator

$$H(t) = \frac{1}{2}p^2 + \frac{1}{2}\omega_0^2 x^2 + \frac{1}{4}\beta x^4 + gx\cos\omega t$$

A naïve expectation

If β and T are sufficiently small,...
The energy remains finite.
⇒ The FM expansion converges?

Summary of our results

- The energy of the system remains finite.
- However, the FM expansion diverges for any non-zero T and β.