YITP-iTHEMS molecule-type international workshop

Potential Toolkit to Attack Nonperturbative Aspects of QFT
-Resurgence and related topics-
Sep 7-25, 2020

Opening remarks & Brief introduction to Resurgence

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@YITP, Kyoto U. & Online 09/07/20

Goals of this workshop

1. **Review** the novel techniques for nonperturbative effects of QFT, focusing on resurgence and the related techniques.

2. **Study** and summarize the very recent results in the techniques.

3. **Raise** and consider questions in the techniques and their physical results.

4. **Propose** their applications to physical problems other than pure QFT.

5. **Discuss** the questions and applications, and produce new works by collaborating with the participants.

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Three Lectures

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Six Talks & Poster Session

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Free Discussion Time

It may be a new manner of holding academic workshops in Covid-19 and Post-Covid-19 eras.

How to join the workshop

1. Lectures, Invited talks, and Short talks will be held in **Zoom**, whose URL has been emailed to all participants.

2. Poster sessions will be held in **Mozilla hubs**. Its URL and how-to-use are shown in the email.

3. Discussion sessions will be also held in **Mozilla hubs**.

All of the important information are shown in **SLACK**, whose URL has been sent to you.
We strongly recommend you to join SLACK ASAP

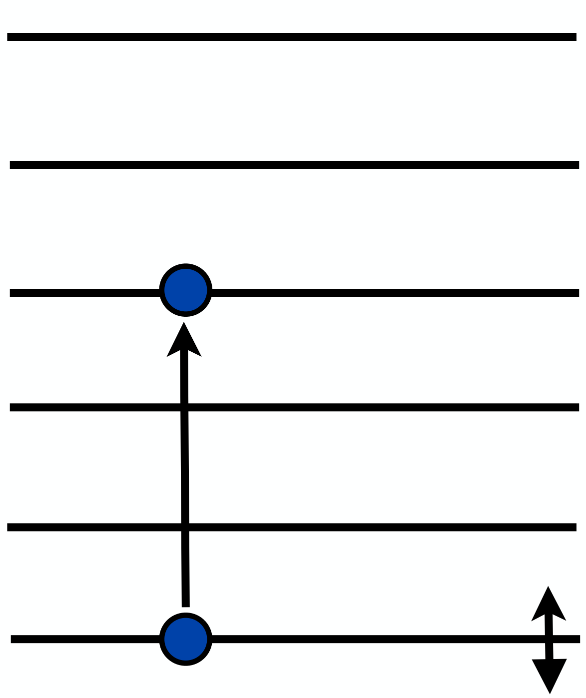
Since some of participants may not be familiar to resurgence theory in quantum physics, we are giving its very brief review and refer to problems to be considered !

See also Prof. Dunne's lecture on 8th

1. Perturbative series and Borel resummation

Perturbative v.s. Non-perturbative analyses in QT

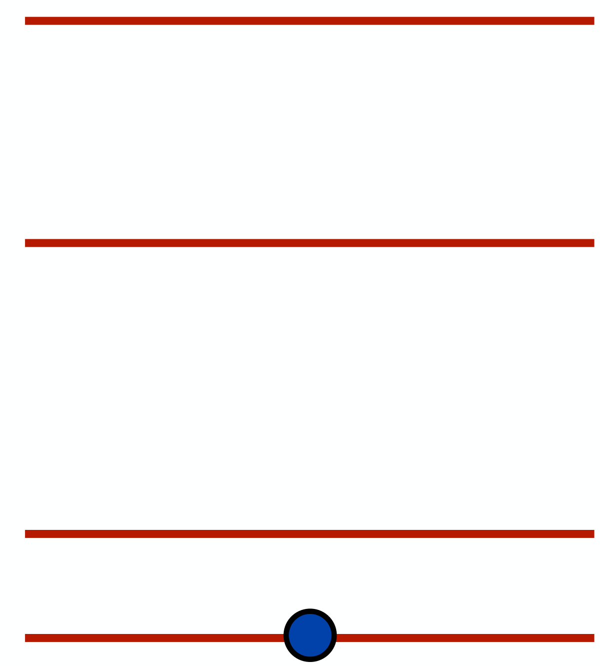
$$H = H_0 + g^2\, H' \qquad g^2 \ll 1$$



Perturbation : quantum fluctuation is calculated as series of g^2 based on eigenstates of H_0

$$E = \sum_{n=0}^{\infty} c_q g^{2q}$$

$$H = H_0 + g^2\, H' \qquad g^2 \sim 1$$



Nonperturbative analysis : it is required to diagonalize whole hamiltonian.

cf.) $E \approx e^{-\frac{A}{g^2}}$

Path integral and Saddle points

$$Z = \int \mathcal{D}\phi \, \exp(-S[\phi]) = \sum_{\sigma \in \text{saddles}} Z_\sigma$$

: stationary points

Trivial (perturbative) saddle

$$Z_0 = \sum_{q=0}^{\infty} a_q g^{2q}$$

Perturbative series

Nontrivial saddles $\frac{\delta S}{\delta \phi} = 0$

$$Z_\sigma \propto e^{-S_{\rm sol}} \sim e^{-\frac{A}{g^2}}$$

Non-perturbative contribution

Relation between Pert. and Non-pert.

$$\sum_{q=0} a_q g^{2q}$$

$$\exp\left[-\frac{A}{g^2}\right]$$

Perturbative series

Non-perturbative contribution

"They are not connected ?
We just have independent contributions ?"

No, it is not correct !

Perturbation and Borel resummation

$$\left[H_0 + g^2 H_{\mathrm{pert}}\right] \psi(x) = E\psi(x)$$

$$P(g^2) = \sum_{q=0}^{\infty} a_q g^{2q}$$ Perturbative series is often divergent factorially $a_q \propto q!$

- Construct an **analytic function** from asymptotic series

➡ Borel resummation：Analytic function which has original perturbative series as asymptotic series

Note that the analytic function is not unique for one asymptotic series.

Perturbation and Borel resummation

$$\left[H_0 + g^2 H_{\text{pert}}\right]\psi(x) = E\psi(x)$$

$$P(g^2) = \sum_{q=0}^{\infty} a_q g^{2q}$$ Perturbative series is often divergent factorially $$a_q \propto q!$$

➡ $$BP(t) := \sum_{q=0}^{\infty} \frac{a_q}{q!} t^q$$ Borel transform

➡ $$\mathbb{B}(g^2) = \int_0^{\infty} \frac{dt}{g^2} e^{-t/g^2} BP(t)$$ Borel resummation

Perturbation and Borel resummation

$$\left[H_0 + g^2 H_{\rm pert}\right] \psi(x) = E\psi(x)$$

$$P(g^2) = \sum_{q=0}^{\infty} a_q g^{2q}$$

Perturbative series is often divergent factorially

$$a_q \propto q!$$

In special cases, Borel resum may give exact results

$$\mathbb{B}(g^2) = \int_0^\infty \frac{dt}{g^2} e^{-t/g^2} BP(t)$$

cf.) x^4 unharmonic oscillator

Perturbation and Borel resummation

$$\left[H_0 + g^2 H_{\rm pert}\right]\psi(x) = E\psi(x)$$

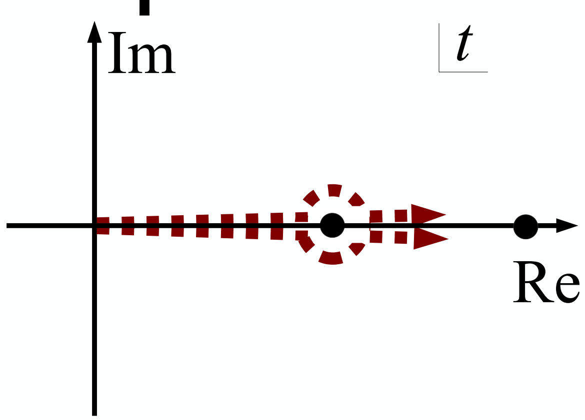
$$P(g^2) = \sum_{q=0}^{\infty} a_q g^{2q}$$

Perturbative series is often divergent factorially

$$a_q \propto q!$$

Borel transform can have singularities on positive real axis

$$BP(t) := \sum_{q=0}^{\infty} \frac{a_q}{q!} t^q$$



$$\Rightarrow \quad \mathbb{B}(g^2 e^{\mp i\epsilon}) = \int_0^{\infty e^{\pm i\epsilon}} \frac{dt}{g^2} e^{-\frac{t}{g^2}} BP(t)$$

Singularities on positive real axis leads to ambiguity

Perturbation and Borel resummation

$$\left[H_0 + g^2 H_{\rm pert}\right] \psi(x) = E\psi(x)$$

$$P(g^2) = \sum_{q=0}^{\infty} a_q g^{2q}$$

Perturbative series is often divergent factorially $\quad a_q \propto q!$

$$\mathbb{B}(g^2 e^{\mp i\epsilon}) = \mathrm{Re}[\mathbb{B}(g^2)] \pm i\mathrm{Im}[\mathbb{B}(g^2)]$$

$$\mathrm{Im}[\mathbb{B}(g^2)] \approx e^{-\frac{A}{g^2}}$$

This should be cancelled by that from non-perturbative contribution!

Non-perturbative effect reappears in perturbative calculation through imaginary ambiguity !

Possible questions to be asked

- A resummation method is not unique. Is there a better resummation formula ?
- Resummation method to get nonperturbative results directly ?
  Costin, Dunne (17)(20)
- Resummation method for the divergent series beyond Gevrey-1?
- Higher-order perturbative series in QFT ?
  cf.) Stochastic perturbation theory

2. Resurgent structure and Trans-series in ODE and Integral

Resurgent structure in ODE

Ecalle (81)

$$\varphi'(z) + \varphi(z) = \frac{1}{z}$$

Formal solutions around $z=\infty$ ⇨ $\Phi_0 = \sum_{q=0}^{\infty} n! z^{-n-1}$ & e^{-z}

perturbative — nonperturbative

Resurgent structure in ODE

Ecalle (81)

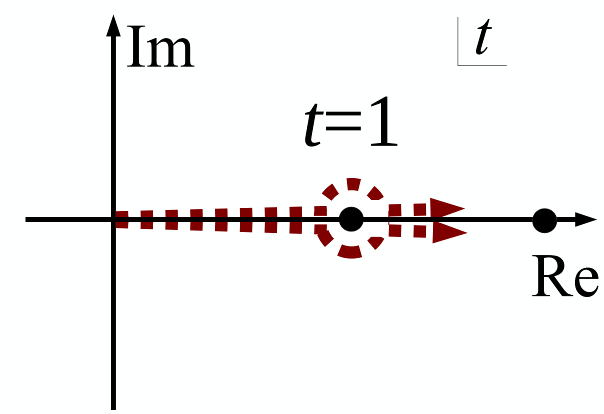
$$\varphi'(z) + \varphi(z) = \frac{1}{z}$$

Formal solutions around $z=\infty$

$$\Phi_0 = \sum_{q=0}^{\infty} n! z^{-n-1} \quad \& \quad e^{-z}$$

perturbative — nonperturbative

$$\mathcal{S}_+\Phi_0(z) - \mathcal{S}_-\Phi_0(z) = 2\pi i e^{-z}$$



resurgent structure!…….. but why?

Resurgent structure in ODE

Ecalle (81)

For review for physicists,
Marino(07),
Aniceto, Schiappa, Vonk(13)
Cherman, Dorigoni, Unsal(14)
Dorigoni(14),
Aniceto, Basar, Schiappa(18)

$$\varphi'(z) + \varphi(z) = \frac{1}{z}$$

Formal solutions around $z=\infty$ ⇨ $\Phi_0 = \sum_{q=0}^{\infty} n! z^{-n-1}$ & e^{-z}

perturbative nonperturbative

1. Solution is expressed as their linear combination = Trans-series

$$\varphi(z;\sigma) = \Phi_0 + \sigma e^{-z} \xrightarrow{\text{Borel-resum}} \mathcal{S}_{\pm}\varphi(z;\sigma) = \mathcal{S}_{\pm}\Phi_0(z) + \sigma e^{-z}$$

2. At arg[z]=0, an appropriate value of σ jumps = Stokes phenomenon

$$\underset{\text{arg[z]=0+}}{\mathcal{S}_{+}\varphi(z;\sigma)} \to \underset{\text{arg[z]=0-}}{\mathcal{S}_{-}\varphi(z;\sigma+\mathfrak{s})}$$

$\mathfrak{s} = 2\pi i$: Stokes constant

3. Continuity of solution leads to resurgent relation between two sectors

$$\mathcal{S}_{+}\varphi(z;\sigma) = \mathcal{S}_{-}\varphi(z;\sigma+\mathfrak{s}) \quad \Rightarrow \quad \mathcal{S}_{+}\Phi_0(z) - \mathcal{S}_{-}\Phi_0(z) = 2\pi i e^{-z}$$

Resurgent structure in ODE

◆Resurgence theory and Alien calculus

$$\varphi(z;\sigma) = \Phi_0 + \sigma e^{-z}$$

- Group action connecting ± Borel resums：Stokes automorphism $\mathfrak{S}$

$$\mathcal{S}_+ = \mathcal{S}_- \circ \mathfrak{S}, \qquad \mathfrak{S} = \exp\left[e^{-z}\Delta\right]$$

- Operator associated with each singularity：Alien derivative Δ

$$[e^{-z}\Delta, \partial_\sigma] = 0 \qquad [e^{-z}\Delta, \partial_z] = 0$$

- Equation bridging alien and standard calculus：Bridge equation

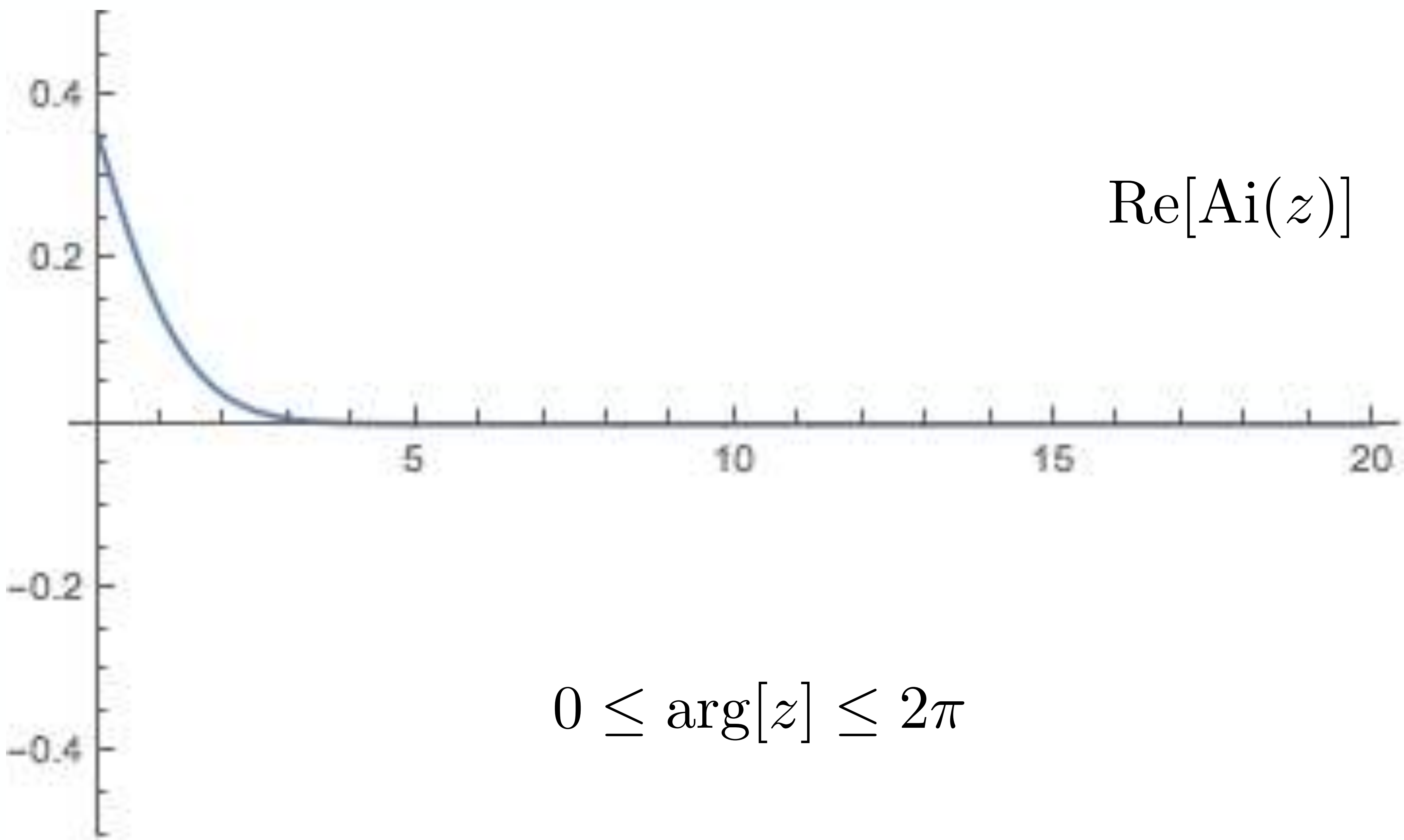
$$e^{-z}\Delta\varphi(z;\sigma) = \mathfrak{s}\,\partial_\sigma\varphi(z;\sigma) \quad \Rightarrow \quad \mathfrak{S}\Phi_0 = \Phi_0 + \mathfrak{s}e^{-z}$$

- Bridge eq. reveals relation between each sector !

$$\mathfrak{S}\varphi(z;\sigma) = \varphi(z;\sigma+\mathfrak{s}) \quad \Rightarrow \quad \mathcal{S}_+\varphi(z;\sigma) = \mathcal{S}_-\varphi(z;\sigma+\mathfrak{s})$$

Ex.) Airy equation $\varphi'' - z\varphi = 0$ (Irregular singularity on $z = \infty$)

$$\varphi = \mathrm{Ai}(z) \approx \underline{e^{-\frac{2}{3}z^{\frac{3}{2}}}\mathcal{S}_{\pm}\sum a_n z^{-\frac{3}{2}n}} + \sigma\, \underline{e^{\frac{2}{3}z^{\frac{3}{2}}}\mathcal{S}_{\pm}\sum b_n z^{-\frac{3}{2}n}}$$



$\mathrm{Re}[\mathrm{Ai}(z)]$

$0 \leq \arg[z] \leq 2\pi$

Resurgent structure in integral

For review for physicists,
Cherman, Dorigoni, Unsal(14)
Tanizaki (14)

In integral, original contour decomposes into **steepest decent contours** (Lefschetz thimbles) associated with **complex saddles**

⇒ **Thimbles associated with distinct saddles have nontrivial relation via Stokes phenomena**

· **Airy integral**

$$\mathrm{Ai}(g^{-2}) = \int_{-\infty}^{\infty} d\phi \exp\left[-i\left(\frac{\phi^3}{3} + \frac{\phi}{g^2}\right)\right]$$

Resurgent structure in integral

Complex saddle contributions in thimble decomposition
(Steepest descent method)

- **Airy integral**

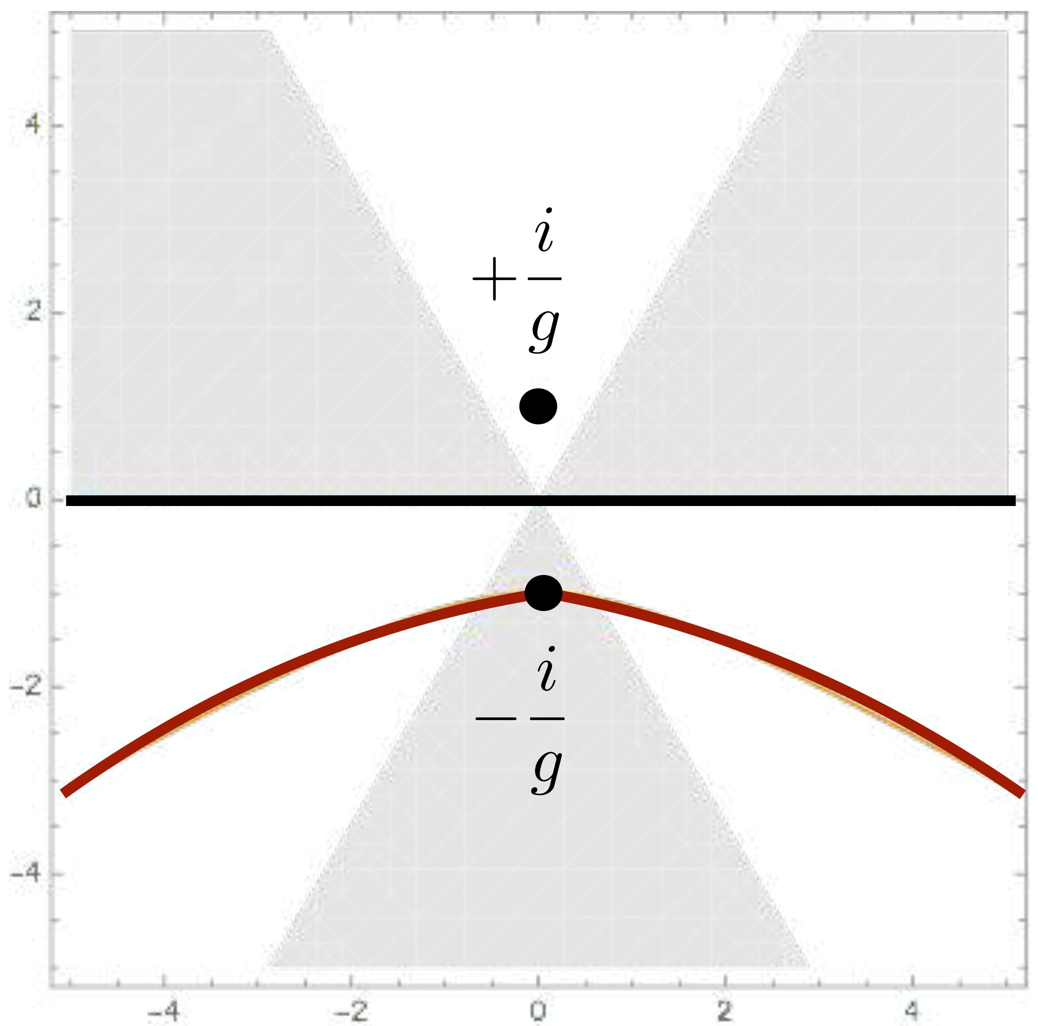
$$\mathrm{Ai}(g^{-2}) = \int_{-\infty}^{\infty} d\phi \exp\left[-i\left(\frac{\phi^3}{3} + \frac{\phi}{g^2}\right)\right]$$

complex saddle points $\phi = \pm \frac{i}{g}$

Steepest descent method :
original contour is decomposed into
thimbles associated with saddle points.

$$\mathcal{C} = \sum_\sigma n_\sigma \mathcal{J}_\sigma$$

Steepest descent contour
= Thimble



$\arg[g^2] = 0+$

Resurgent structure in integral

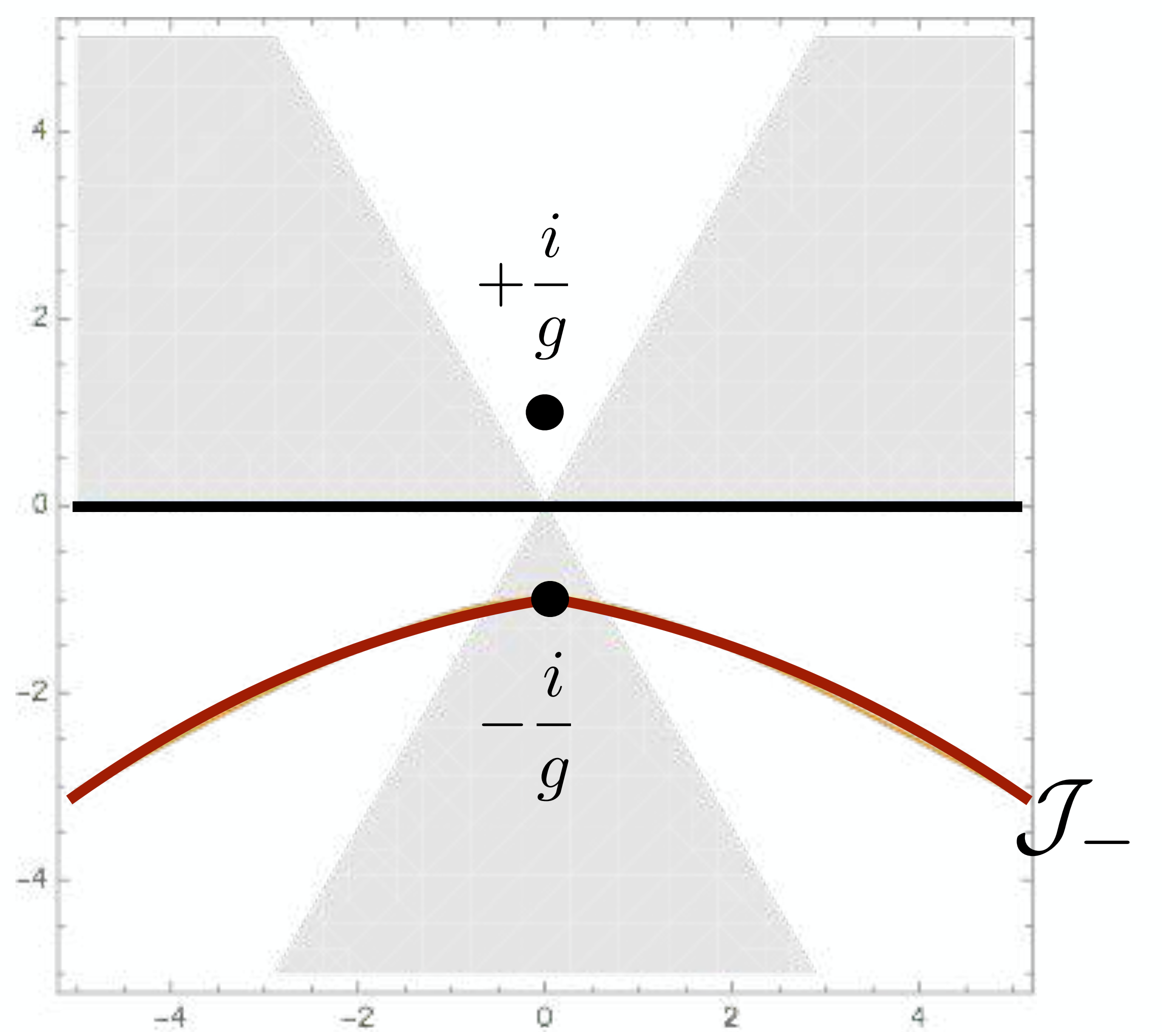
Complex saddle contributions in thimble decomposition
(Steepest descent method)

- **Airy integral**

$$\mathrm{Ai}(g^{-2}) = \int_{-\infty}^{\infty} d\phi \exp\left[-i\left(\frac{\phi^3}{3} + \frac{\phi}{g^2}\right)\right]$$

- $\mathcal{J}_\sigma$ $\quad \mathrm{Im}[S] = \mathrm{Im}[S_0]$, $\mathrm{Re}[S] \leq \mathrm{Re}[S_0]$ **Thimble**
- $n_\sigma = \langle \mathcal{K}_\sigma, \mathcal{C} \rangle$ **Intersection number of dual thimble $\mathcal{K}$ and original contour**

⇨ $$\mathcal{C} = \sum_\sigma n_\sigma \mathcal{J}_\sigma$$



$$\arg[g^2] = 0+$$

Resurgent structure in integral

Complex saddle contributions in thimble decomposition (Steepest descent method)

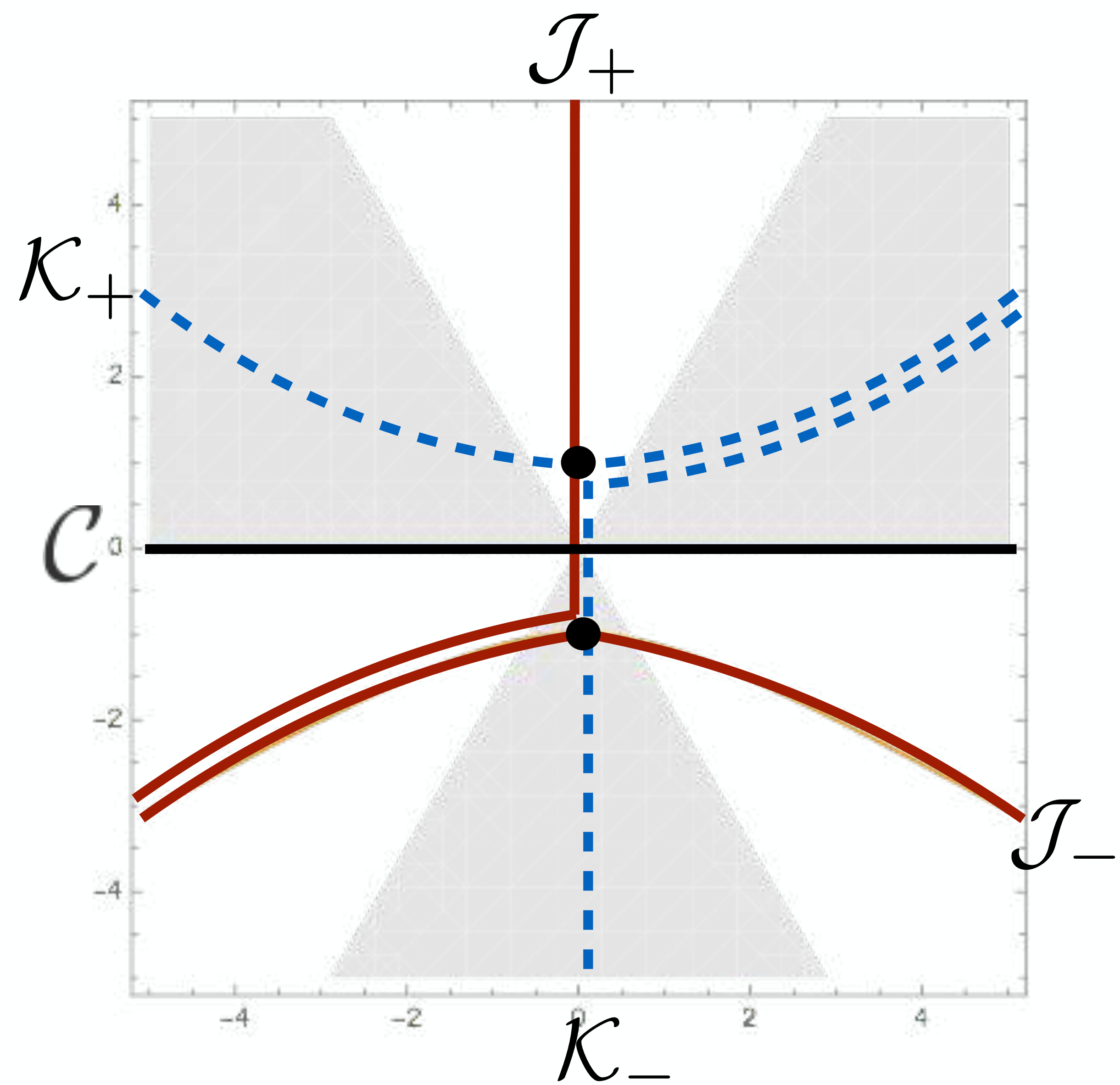
- **Airy integral** $\arg[g^2] = 0+$

$$n_+ = \langle \mathcal{K}_+, \mathcal{C} \rangle = 0$$

$$n_- = \langle \mathcal{K}_-, \mathcal{C} \rangle = 1$$

$$\mathcal{C} = \sum_\sigma n_\sigma \mathcal{J}_\sigma \Rightarrow \boxed{\mathcal{C} = \mathcal{J}_-}$$

valid decomposition till $\arg[g^2] = \frac{2\pi}{3} -$



$\arg[g^2] = 0+$

Resurgent structure in integral

Complex saddle contributions in thimble decomposition (Steepest descent method)

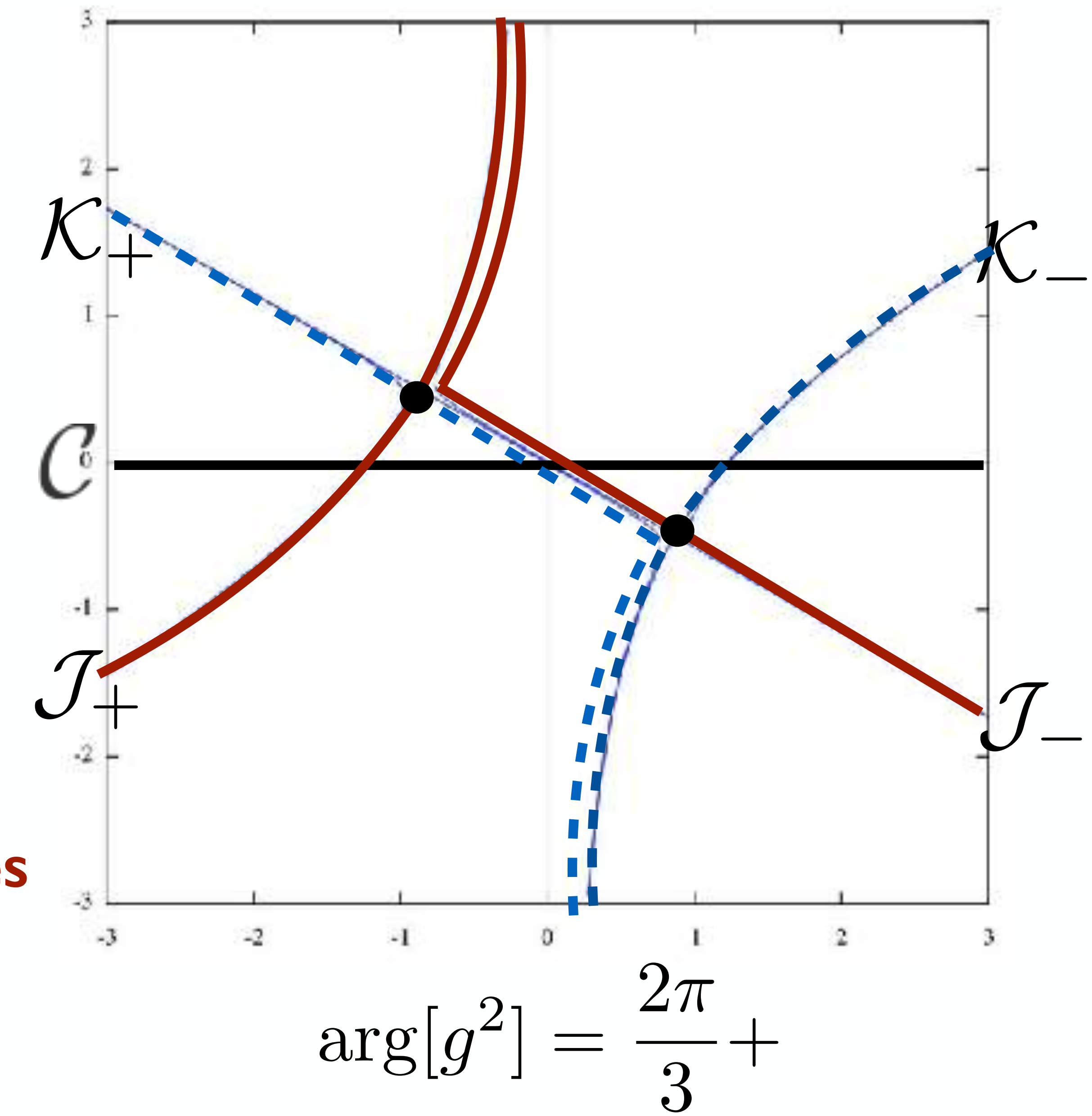
- **Airy integral** $\arg[g^2] = \frac{2\pi}{3} +$

$$n_+ = \langle \mathcal{K}_+, \mathcal{C} \rangle = 1$$

$$n_- = \langle \mathcal{K}_-, \mathcal{C} \rangle = 1$$

$$\mathcal{C} = \sum_\sigma n_\sigma \mathcal{J}_\sigma \Rightarrow \boxed{\mathcal{C} = \mathcal{J}_- + \mathcal{J}_+}$$

Stokes phenomenon : at special arg[g^2], thimble decomposition discretely changes



Resurgent structure in integral

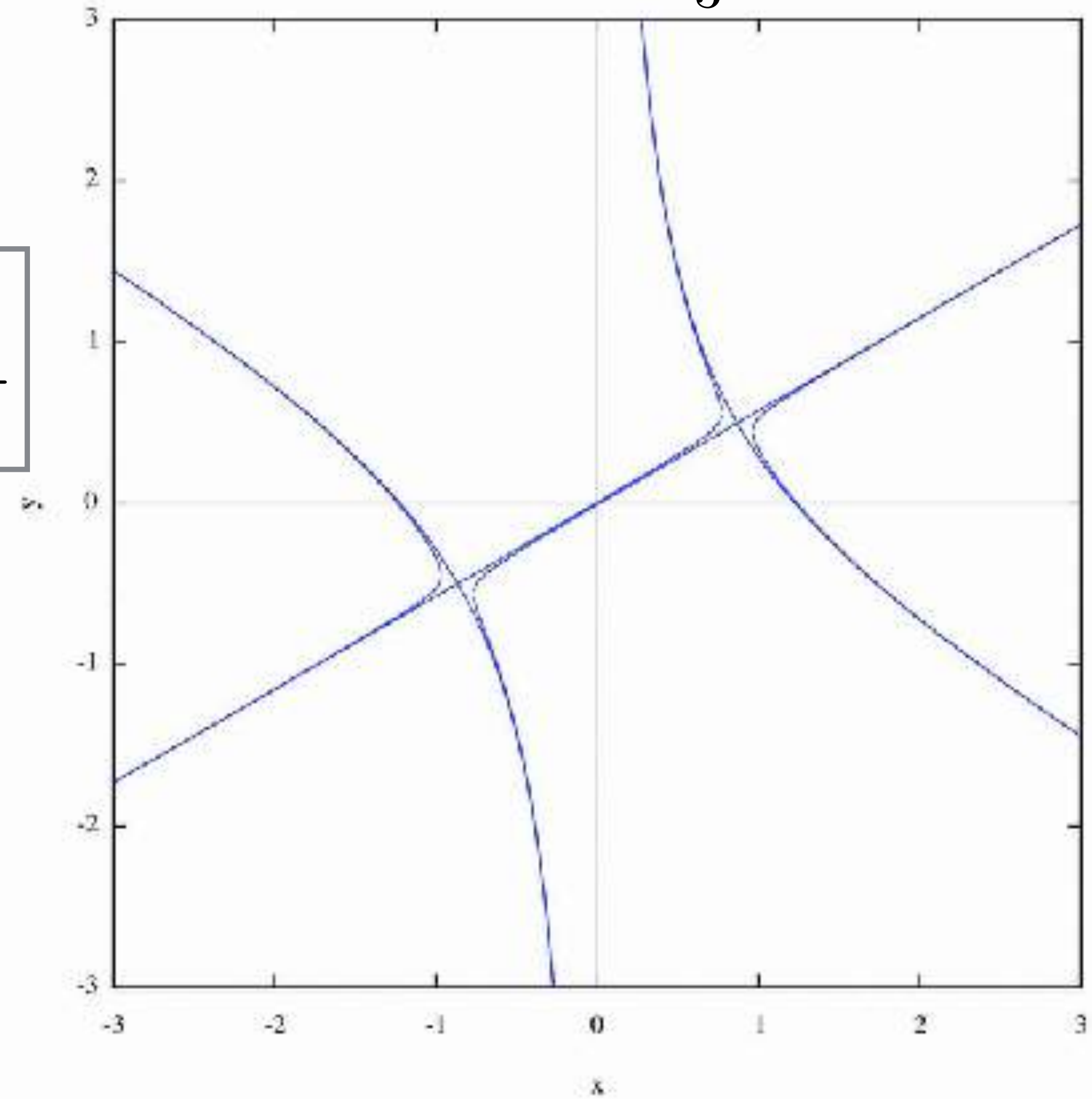
Complex saddle contributions in thimble decomposition (Steepest descent method)

- Airy integral

$$\arg[g^2] = \frac{2\pi}{3}^{-} \qquad\qquad \arg[g^2] = \frac{2\pi}{3}^{+}$$

$$\boxed{\mathcal{C} = \mathcal{J}_-} \quad \Leftrightarrow \quad \boxed{\mathcal{C} = \mathcal{J}_- + \mathcal{J}_+}$$

$\arg[g^2] = -\frac{2\pi}{3} \to \pi$



* Thimble decomposition is discretely changed at Stokes line.
* Airy function is continuous even at the Stokes line.

$$\mathcal{J}_-\left[\frac{2\pi}{3}^{-}\right] = \mathcal{J}_-\left[\frac{2\pi}{3}^{+}\right] + \mathcal{J}_+$$

Resurgent structure in integral

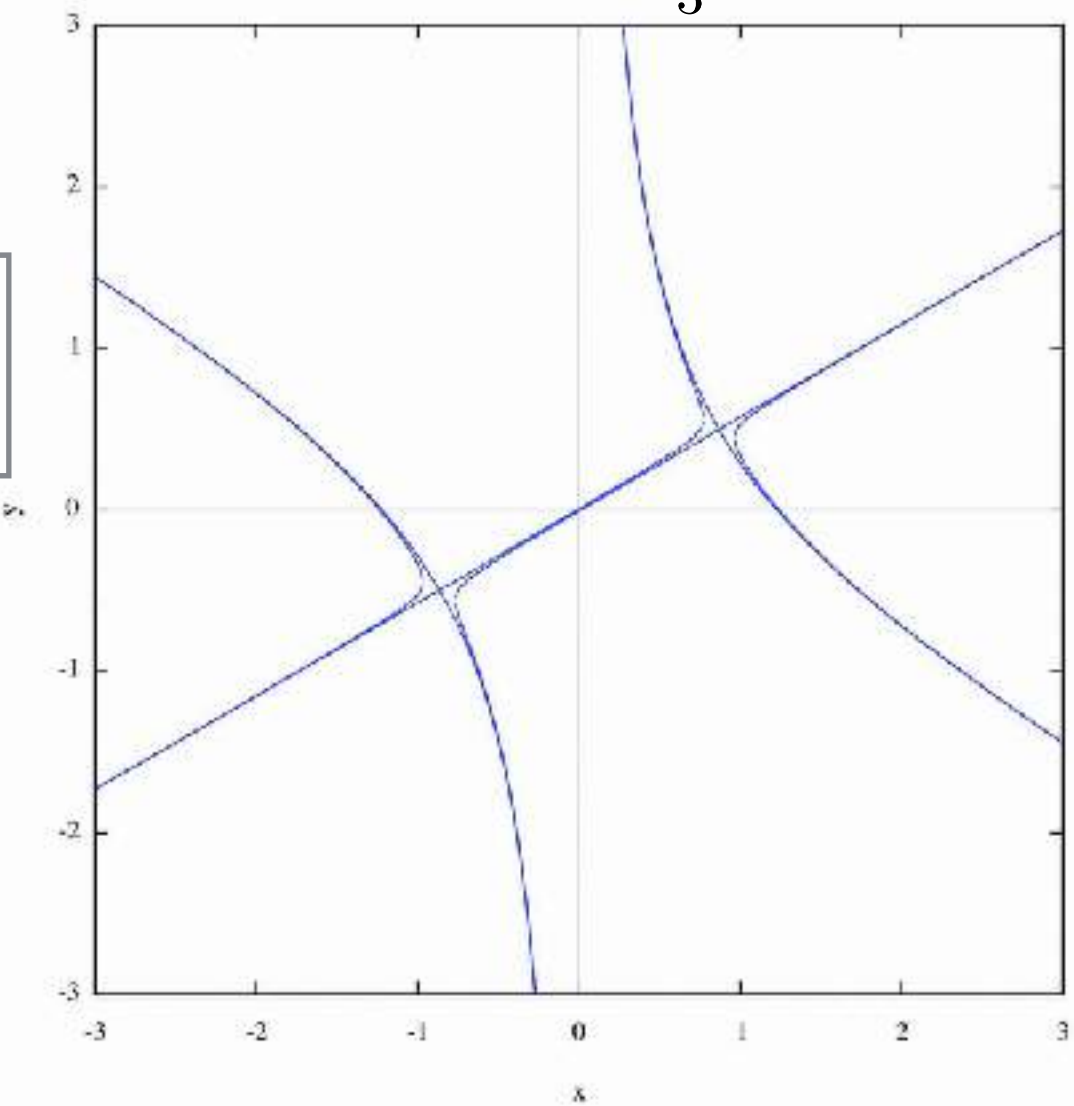
Complex saddle contributions in thimble decomposition (Steepest descent method)

- Airy integral

$$\arg[g^2] = \frac{2\pi}{3}- \qquad\qquad \arg[g^2] = \frac{2\pi}{3}+$$

$$\mathcal{C} = \mathcal{J}_- \quad \Leftrightarrow \quad \mathcal{C} = \mathcal{J}_- + \mathcal{J}_+$$

$$\arg[g^2] = -\frac{2\pi}{3} \to \pi$$



* Thimble decomposition is discretely changed at Stokes line.

* Airy function is continuous even at the Stokes line.

Two thimbles have resurgent relation via ambiguity due to Stokes phenomena !

Resurgent structure in ODE and integral

$$\mathcal{S}_{+}\Phi_0(z) - \mathcal{S}_{-}\Phi_0(z) \approx \mathfrak{s} e^{-Az} \mathcal{S}\Phi_1(z)$$

Perturbative imaginary ambiguity

Non-perturbative effect

Resurgent structure is expected to be in quantum theory, thus perturbative series could include nonpert. information !

⇒ **In a certain class of quantum theories, we may be able to derive non-perturbative result from perturbation**

Possible questions to be asked

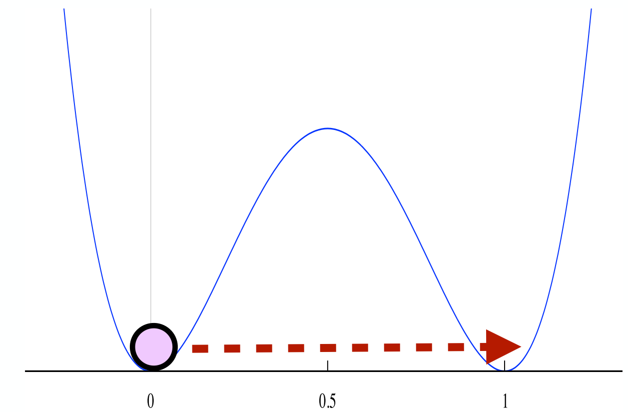
- Thimble decomposition is possible in QFT path integral ?
- Resurgent functions in resurgence theory for ODE can be extended ?
- How can we figure out the intersection number ?
- Resurgent structure in partial differential equations ?

3. Resurgent structure in quantum theory

Example in Double-well QM

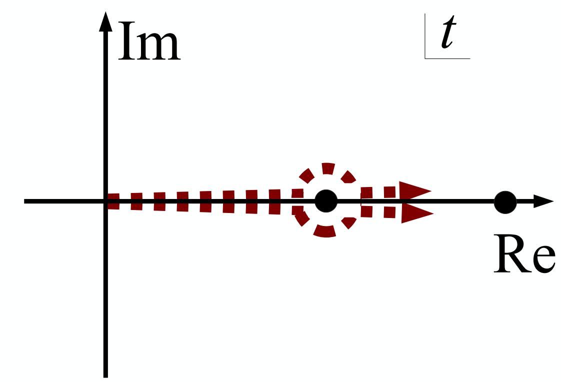
Bender-Wu(73)
Bogomolny(77) Zinn-Justin(81)

- Perturbation in Double-well QM



$$H = \frac{p^2}{2} + \frac{x^2(1-gx)^2}{2}$$

$$E_{0,pert} = \sum_{q=0} a_q g^{2q} \qquad a_q = -\frac{3^{q+1}}{\pi} q! \qquad (\, q \gg 1 \,)$$



$$\Rightarrow \quad BP_{pert}(t) = \frac{1}{\pi} \frac{1}{t - 1/3}$$

Singularity on positive real axis

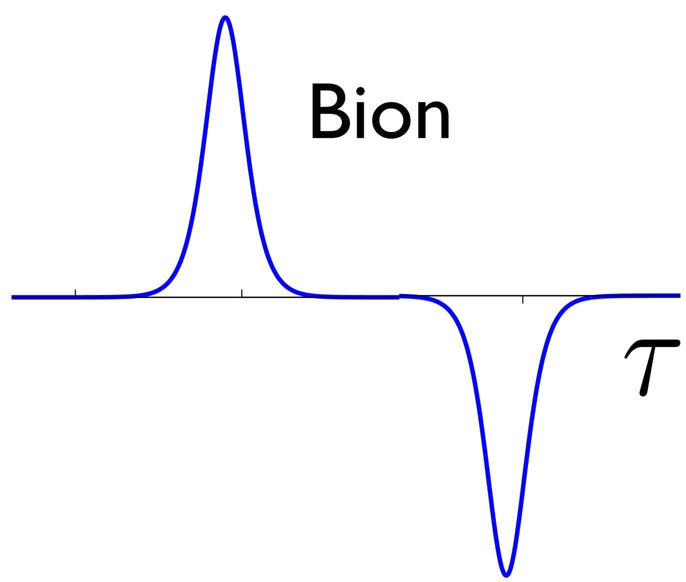
$$\Rightarrow \quad \mathrm{Im}[\mathbb{B}_{pert}(g^2 e^{\mp i\epsilon})] = \mathrm{Im}\left[\int_0^{\infty e^{\pm i\epsilon}} \frac{dt}{g^2} \frac{e^{-\frac{t}{g^2}}}{\pi} \frac{1}{t-1/3}\right] = \mp \frac{e^{-\frac{1}{3g^2}}}{g^2}$$

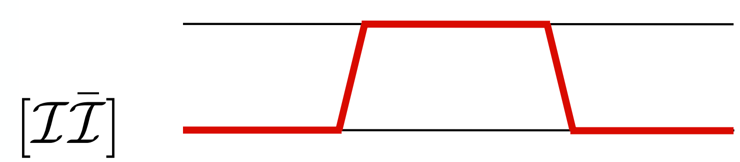
Instanton solution $\quad x(\tau) = \frac{1}{2g}\left(1 + \tanh \frac{\tau - \tau_{\mathcal{I}}}{2}\right)$ $\qquad S_I = \frac{1}{6g^2}$

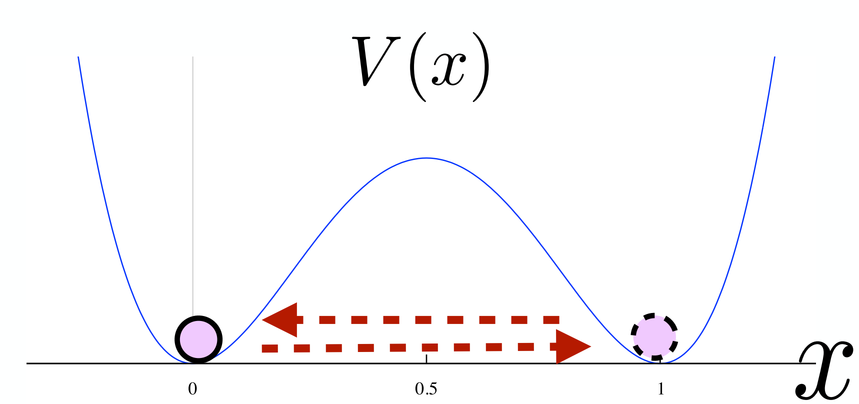
twice

Instanton-antiinstanton configuration = Bion

$$x_{\mathcal{I}\bar{\mathcal{I}}} = \frac{1}{2g}\left[\tanh\frac{\tau-\tau_{\mathcal{I}}}{2} - \tanh\frac{\tau-\tau_{\bar{\mathcal{I}}}}{2}\right]$$







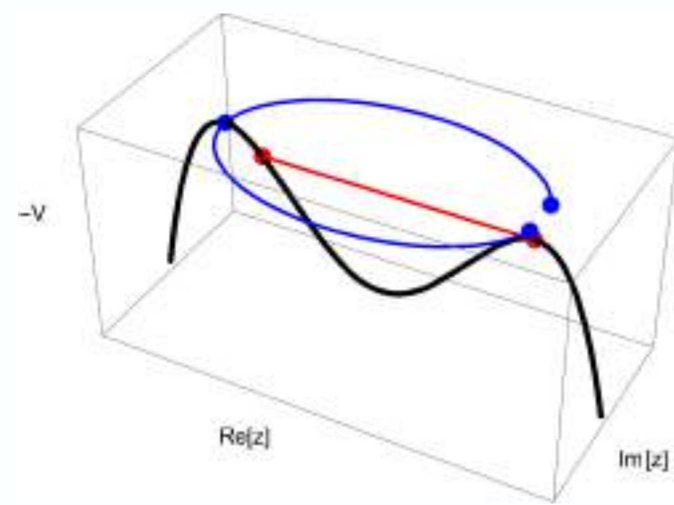
$$E_b \approx -\lim_{\beta\to\infty}\frac{1}{\beta}\frac{Z_b}{Z_0} = \frac{e^{-2S_I}}{\pi g^2}\left[\gamma + \log\frac{2}{g^2} \pm i\pi\right]$$

cancels the imaginary ambiguity $\mp\frac{e^{-2S_I}}{g^2}$ in perturbation

Complex bion solution and its contribution

Behtash, Dunne, Schafer, Sulejmanpasic, Unsal(15)
For other models including CPN-1 QM,
See Fujimori, et.al. (16)(17)

$$x \to z = x + iy$$



$$\frac{d^2 z}{d\tau^2} = \frac{\partial V}{\partial z}$$

- Complex bion solutions

$$z_{cb}(\tau) = z_1 - \frac{(z_1 - z_T)}{2} \coth \frac{\omega \tau_0}{2} \left[\tanh \frac{\omega(\tau + \tau_0)}{2} - \tanh \frac{\omega(\tau - \tau_0)}{2} \right] \qquad z_T,\ \tau_0 \in \mathbb{C}$$

- Contribution from complex bion

$$E_{cb} = \frac{e^{-\frac{1}{3g^2}}}{\pi g^2} \left(\frac{g^2}{2} \right)^{\epsilon} \left[-\cos(\epsilon \pi) \Gamma(\epsilon) \pm \frac{i\pi}{\Gamma(1-\epsilon)} \right]$$

The imaginary ambiguity cancels that from perturbative series

For precise summation of bion contributions,
see Sueishi (19).

Possible questions to be asked

- Non-trivial Borel singularities always correspond to complex solutions ?

- How can we treat systems with fermions ?
  By projection? By integrating out?

- Can we clarify relation of Stokes phenomena in semi-classical calculation and Exact WKB analysis ?

  Sueishi-san's talk on 18th

- The same structure exists in quantum field theory ?

Examples in QFT and Matrix model

- 2D sigma model with compactification

  Dunne, Unsal(12) TM, Nitta, Sakai(14)(15) Fujimori, et.al.(18) Ishikawa, et.al. (19)

- 3D Chern-Simons theory (with matter)

  Gukov, Marino, Putrov(16-) Honda(16) Fujimori, Honda, Kamata, TM, Sakai(18)

- 4D N=2 Super Yang-Mills on S^4

  Schiappa, Marino, Aniceto(13~) Honda(16)

- Matrix models & Topological String Theories

  Marino(07~) Marino, Schiappa, Weiss(09), Hatsuda, Dorigoni(15)

Prof. Dunne's lecture on 8th
Hatsuda-san's talk on 18th
Honda-san's talk on 25th
Yoda-san's poster on 11th

4. Resurgent structure in gauge theory

Infrared renormalon in QCD

't Hooft(79)

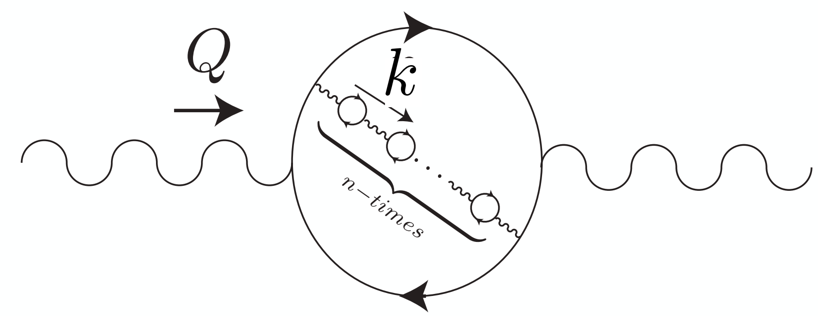
In asymptotically free QFT, another source of Im ambiguity exists.

◆Adler function and renormalon

$$D(Q^2) = 4\pi^2 \frac{d\Pi(Q^2)}{dQ^2}$$

$$D(Q^2) = \sum_{n=0}^{\infty} \alpha_s \int_0^\infty d\hat{k}^2 \frac{F(\hat{k}^2)}{\hat{k}^2} \left[\beta_0 \alpha_s \log \frac{\hat{k}^2 Q^2}{\mu^2} \right]^n$$

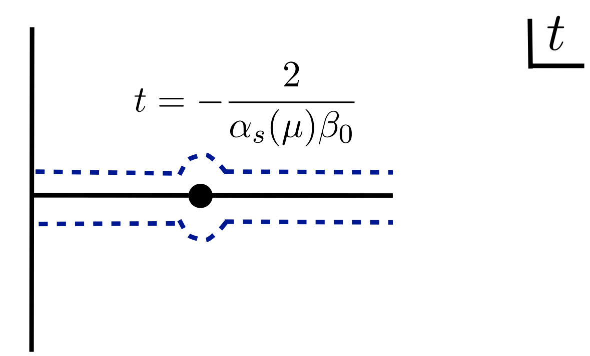
$$\approx \alpha_s \sum_n \left(-\frac{\alpha_s \beta_0}{2} \right)^2 n! + \text{UV contr.}$$



$$\Rightarrow \quad BP(t) = \alpha_s(\mu) \sum_n \left(-\frac{\alpha_s(\mu)\beta_0 t}{2} \right)^n = \frac{\alpha_s(\mu)}{1 + \alpha_s(\mu)\beta_0 t/2}$$

$$\Rightarrow \quad t = -\frac{2}{\alpha_s(\mu)\beta_0}$$

Singularity on real axis ($\beta_0 = -\frac{11N_c - 2N_f}{12\pi}$)



$$\Rightarrow \quad \mathbf{B}(\alpha_s) = \text{Re}\mathbf{B} \pm \frac{i\pi}{\beta_0} e^{\frac{2}{\alpha_s \beta_0}} \approx \left(\frac{\Lambda_{QCD}}{Q} \right)^4$$

could be related to QCD scale
and low-energy physics
→ IR renormalon

QCD(adj.) on $R^3 \times S^1$

Argyres-Unsal (12)

Conjecture : Neutral bion is identified as IR-renormalon

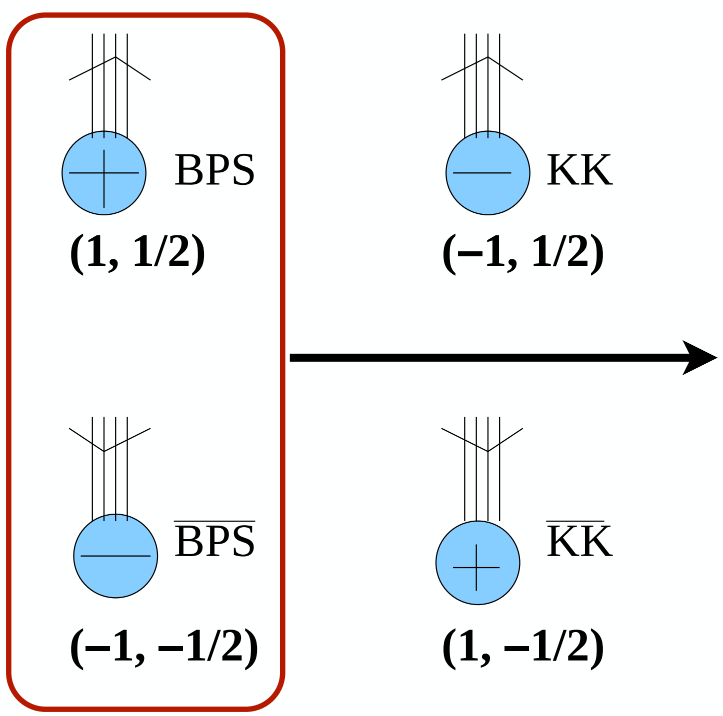
- Non-trivial Polyakov-loop holonomy for small S1

$$P = \mathrm{diag}[1, e^{2\pi i/N}, e^{4\pi i/N}, \cdots, e^{2\pi(N-1)i/N}] \qquad (P^N = \mathbf{1})$$

⇒ $1/N$ fractional instantons ($Q=1/N$, $S=S_I/N$)

For YM on R3×S1 with 't Hooft twist see also Yamazaki, Yonekura(17), Itou(19). For confinement via magnetic bions, see Unsal(07).

cf.)SU(2)



Neutral bion = IR-renormalon
???

Prof. Unsal's lecture on 10th Prof. Cherman's lecture on 22th Morikawa-san's talk on 24th Yamazaki-san's talk on 25th

$CP^{N\text{-}1}$ model on $R^1 \times S^1$

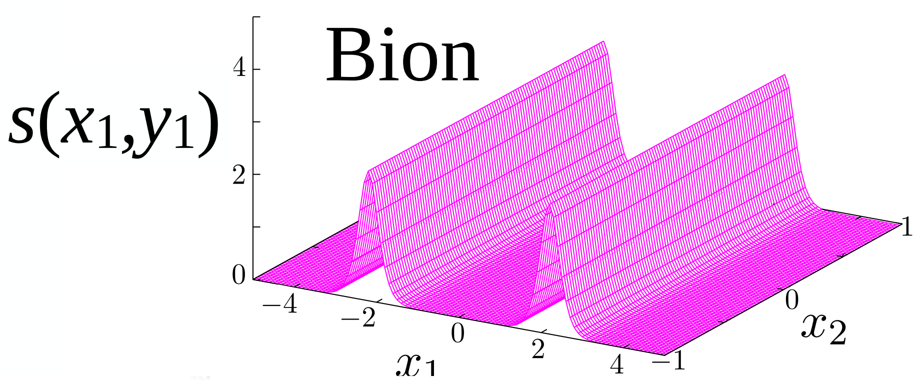
Dunne-Unsal (12)
Fujimori,et.al. (16)(17)(18)

Conjecture : Complex bion is identified as IR-renormalon

- Z_N-twisted boundary condition on S^1 direction

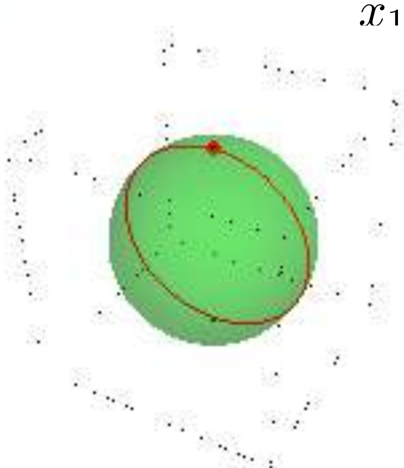
$$\vec{z}(x^1, x^2 + L) = \Omega \vec{z}(x^1, x^2) \qquad \Omega = \mathrm{diag}(1, \omega, \ldots, \omega^{N-1})$$

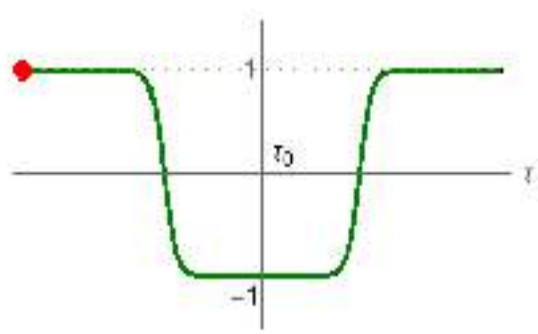
$\Rightarrow$ 1/N fractional instantons (Q=1/N, S=S_I/N)



Bion imaginary ambiguity seems consistent with IR-renormalon.
But, is it related to renormalon on R^2 ?
There have been arguments.

Ishikawa, et.al. (19) Yamazaki, Yonekura (19) etc….





$\Rightarrow$

Fujimori-san's &
Morikawa-san's talks on 24th
Yamazaki-san's talk on 25th

Adiabatic continuity in compactified QFT

Adiabatic continuity conjecture: Vacuum structure & Z_N symmetry persist during Z_N-twisted compactification

If adiabatic continuity exists,
the resurgent structure on compactified spacetime
has implications on decompactified spacetime.

- 2D Z_N-twisted model — Sulejmanpasic (16) Tanizaki, TM, Sakai (17) Hongo, Tanizaki, TM(18) TM, Tanizaki, Unsal(19) Fujimori, et.al. (20)
- QCD(adj.) + Z_N-twisted quarks — Cherman, Schafer, Unsal (16), (See also Iritani, Itou, TM(15))
- YM with ’t Hooft twist — Yamazaki, Yonekura (17), Itou (19)

Prof. Unsal’s lecture on 10th
Prof. Cherman’s lecture on 22th
Fujimori-san’s talk on 24th
Chen-san’s, Itou-san’s, Tanizaki-san’s posters on 11th

5. Further applications of resurgence theory

Further applications

- Condensed matter physics, High-T_c superconductor
- Hydrodynamics, Fluid dynamics
- Schwinger mechanism
- Quantization conditions via exact-WKB, TBA equations
- Cosmology & Astrophysics
- Phase transition

Prof. Dunne's lecture on 8th
Hatsuda-san's talk on 18th
Honda-san's talk on 25th
Posters by Arraut, Du, Harris, Imaizumi, Shimazaki, Yoda

Goals of this workshop

1. **Review** the novel techniques for nonperturbative effects of QFT, focusing on resurgence and the related techniques.

2. **Study** and summarize the very recent results in the techniques.

3. **Raise** and consider questions in the techniques and their physical results.

4. **Propose** their applications to physical problems other than pure QFT.

5. **Discuss** the questions and applications, and produce new works by collaborating with the participants.